# EVOLUTION EQUATIONS OF THE $p(x,t)$-LAPLACE TYPE

SERGEY SHMAREV

The presentation is an overview of the recent results in the study of the evolution $p(x,t)$-Laplace equation,

$$\partial_t u - \operatorname{div}\left(a(x,t,u)|\nabla u|^{p(x,t)-2}\nabla u\right) + \mathcal{F}(x,t,u,\nabla u) = 0,$$

and the anisotropic equations

$$\partial_t u - \sum_{i=1}^{n} D_i\left(a_i(x,t,u)|D_i u|^{p_i(x,t)-2} D_i u\right) + \mathcal{F}(x,t,u,\nabla u) = 0$$

with given variable exponents $p$ and $p_i$. We consider the homogeneous Dirichlet problem in a cylinder $Q_T = \Omega \times (0,T]$ where $\Omega \subset \mathbb{R}^n$ is a bounded domain.

The following issues are discussed.

- The function spaces: definitions and basic properties of Orlicz-Lebesgue and Orlicz-Sobolev spaces $L^{p(\cdot)}(\Omega)$ and $W_0^{1,p(\cdot)}(\Omega)$, parabolic spaces, approximation by smooth functions, condition of log-continuity of the exponents.
- Different notions of weak solution.
- Existence theorems in dependence on the regularity of the exponents $p_i(x,t)$.
- Classes of uniqueness.
- Localization properties of weak solutions: finite speed of propagation of disturbances from the data, vanishing in a finite time.
- Localization in the limit cases: vanishing in a finite time of solutions of the eventually linear equations.
- Localization effects in solutions of the anisotropic equations: directional localization and nonpropagation of disturbances, simultaneous localization in space and time.
- Blow-up of solutions of equations with variable and /or anisotropic nonlinearity.

University of Oviedo, Department of Mathematics
*E-mail address*: shmarev@uniovi.es